Vibration Analysis of Railway Tracks Forced by Distributed Moving Loads

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Abstract

The purpose of this study was to develop a theoretical model to analyze the vibration of finite railways forced by distributed moving loads. The vibration characteristics of compliantly supported beam utilizing compressional damping model were investigated through the Rayleigh-Ritz method. The distributed moving load was analyzed as the cross correlation function on railways. This allowed the use of statistical characteristics for simulation of the moving train wheels on the rail. The results showed there is a critical velocity inducing resonant vibration of the rail. The mass spring resonance from the rail fastening systems exhibited significant influence on the resulting vibration response. In particular, the effect of the viscoelastic core damping was investigated as an efficient method for minimizing rail vibration. The decrease of the averaged vibration and rolling noise generation by the damping core was maximized at the mass-stiffness-mass resonance frequency.

Keywords: Rayleigh-ritz method, Rolling noise, Sandwich panel

1. Introduction

A complex structures composed of multi-layer panels is used to dissipate vibration energy in many engineering applications. Damping material inserted as core in complex structures reduces resonant responses and noise radiation. Damping treatments have advantages on costs and maintenance compared to other vibration reduction methodologies. The modal characteristics of entire systems are determined from interaction between each layer for the complex structures. Moving load caused by train induces track vibration and is an important source of environmental noise on the nearby residential areas. Theoretical model for vibration of the complex structure with core damping material is required for optimal design for decreased structural vibration in the frequency range of noise pollution.

Sandwich models to analyze vibration of complex structures have been studied for decades. Rao and Nakra [1] proposed the governing equation required for analyzing vibration of sandwich panel with viscoelastic layer between two rigid panels. Douglas and Yang [2] analyzed frequency which influences translational deformation of viscoelastic layer in sandwich beam and compared different vibration models. The translational vibration was dominant as the thickness of the upper stiff panels increases. [3] Although the multi-layer construction is common in railway tracks, the numerical method for investigation the random vibration in audio frequency ranges has not been proposed, especially on analysis of the effects of the core dynamic properties.

In this paper, rail-slab complex structure is analyzed by the compressional damping model. The influence of the core damping material on the vibration characteristics of the rail structures was analyzed through the Rayleigh-Ritz method. The distributed moving force using statistical characteristics was utilized for simulation of loads from operating trains to the rail structures. The influence of the core damping layer on resulting vibration was investigated.

2. Free Vibration Analysis of a Rail on a Slab

2.1 Rayleigh-Ritz methods

As shown in Fig. 1 the numerical approaches for
understanding the railway track vibration were performed. The moving loads from the operating train are applied to each beam. The Rayleigh-Ritz method was applied for the vibration analysis of the track model. Shear deformation of the core damping materials between beam and plate was ignored. This assumption is typically applied when the bending stiﬀnesses of the beam and the plate are large and the modulus of the damping material is small. In addition, changes in the dynamic properties of the damping material which typically shows variation of 5-10% with frequency was ignored. Displacements of each structural element were assumed as follows using the N trial functions for the beam and 2N trial functions for the plate. Lagrangian equations were applied to these equations as

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\alpha}_{mn}} \right) - \frac{\partial L}{\partial \alpha_{mn}} = 0, \quad mn = 1, 2, \ldots, N^2 \tag{1a}
\]

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\beta}_{mn}} \right) - \frac{\partial L}{\partial \beta_{mn}} = 0, \quad mn = 1, 2, \ldots, N^2 \tag{1b}
\]

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\lambda}_{mn}} \right) - \frac{\partial L}{\partial \lambda_{mn}} = 0, \quad mn = 1, 2, \ldots, N^2 \tag{1c}
\]

where \( L = T - V_p - V_k \) is the system Lagrangian. The displacements of the plates \((w_1, w_2)\) and the beams \((w_3, w_4)\) represented as the product of trial function and normal coordinate \((\alpha_{mn}, \beta_{mn}, \lambda_{mn})\):

\[
\begin{bmatrix}
    w_1 \\
    w_2 \\
    w_3 \\
    w_4
\end{bmatrix} =
\begin{bmatrix}
    \phi_{mn,1} & 0 & 0 & 0 \\
    0 & \phi_{mn,2} & 0 & 0 \\
    0 & 0 & \phi_{mn,3} & \phi_{mn,4}
\end{bmatrix}
\begin{bmatrix}
    \alpha_{mn} \\
    \beta_{mn} \\
    \lambda_{mn}
\end{bmatrix}.
\tag{2}
\]

The kinetic and potential energies of the system were calculated for the application of Rayleigh-Ritz method. The kinetic energy was calculated as

\[
T = \int_a^b \rho \left( \frac{\partial w}{\partial t} \right)^2 \, dx + \int_a^b \rho_h \left( \frac{\partial w}{\partial t} \right)^2 \, dx + \int_a^b \rho_h \left( \frac{\partial w}{\partial t} \right)^2 \, dx
\tag{3}
\]
3.2 Calculation of average vibration response from distributed moving loads

The vibration response by the distributed moving load was analyzed through the cross correlation function. The cross correlation function for the translational vibration responses of the railway track was defined as follows using the transfer response function of forces, \( S_{pp} \), acting to each plates as

\[
S_{yy}(r_1, r_2, \omega) = \int_0^1 \int_0^1 [H^p(r_1, s_1, \omega)H(r_2, s_2, \omega)S_{pp}(s_1, s_2, \omega)]ds_1ds_2. \tag{9}
\]

where \( r_1 = (x_{p1}, y_{p1}, x_{s1,1}, y_{s1,1}, x_{s1,2}) \) and \( r_2 = (x_{p2}, y_{p2}, x_{s2,1}, y_{s2,1}, x_{s2,2}) \). Similarly, the variables \( s_1 \) and \( s_2 \) are positions of force acting on the rail. Spatial average vibration is represented as follows

\[
\int_{0}^{0} S_{yy}(r_1, r_2, \omega)d\omega = N_2A \sum_{j=1}^{N+2N} H_j^p(\omega)H_j(\omega)V_{j,1,2,3}S_{pp}(\omega). \tag{10}
\]

The wavenumber components of the force is represented as

\[
S_{pp}(k, x_1, x_2, \omega) = \frac{A^2}{2} e^{-j(\omega V + k)(x_2 - x_1)}. \tag{11}
\]

From equation (10), the vibration response from arbitrary moving loads is obtained by superposing responses of impulse forces, which has wavenumber \( k \). As a result, when the load moving with velocity \( V \) is applied to the railway, loads applied to each plate and beam are analyzed.

3.3 Response of rail-slab model by moving load

When the load moves on the railway, the responses of the rail and the slab were calculated. With the rail excited by the wheel of the train, it was assumed that the external excitation was induced by the pressing force at the contacting area. The mechanical properties of the rails assuming 41GPU rail and concrete slabs applied for the numerical analysis is summarized in Table 1. Analysis was conducted with \( N = 30 \) to warrant the convergence up to 2000 Hz. The speed of the load was increased from 2 m/s by 0.5 m/s to investigate the effects of moving velocity.

The averaged vibration response increases significantly with the increasing velocity of the load as shown in Fig. 2. The frequency of maximum vibration response also increased with a constant slope depending on the velocity. When the velocity of the maximum vibration response and the natural frequency of the railways are identical, it resulted in very resonant response. For the rail, the resonances at 800 Hz and 1200 Hz were observed. The vibration magnitude of the rail was much larger than those of the slab, as expected from direct application of the force on the rail.

<table>
<thead>
<tr>
<th>Table 1 Applied properties of rail and slab</th>
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<tbody>
<tr>
<td>Properties</td>
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<tr>
<td>Length((a))</td>
</tr>
<tr>
<td>Width((b))</td>
</tr>
<tr>
<td>Thickness((h))</td>
</tr>
<tr>
<td>Density((\rho))</td>
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<tr>
<td>Young’s Modulus ((E))</td>
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<td>Loss factor ((\eta))</td>
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<td>Moment of inertia ((I))</td>
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<td>(k_f)</td>
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<td>(k_s)</td>
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</table>

Fig. 2 Averaged velocity response of (a) the rail and (b) the slab with moving load speed. The vibration response increased with the increasing moving velocity.
3.4 Change of structural vibration characteristics by core damping materials

The rail pads in railway tracks are used to reduce vibration and noise by moving train wheels on the rail and to prevent rails from excessive corrugations. The system dynamic characteristics are influenced by inserted rail pads. The effect of the core damping material on the complex structure is required to be understood. Fig. 3 shows the velocity response of the rail and slab with the change in the stiffness of the core damping material. Two harmonic forces of 5 kN were assumed to move with the velocity of 10 m/s and 0.03 m apart. When the stiffness increased from 80 MPa to 140 MPa, the natural frequency changed, and consequently, the frequency range of maximum vibration response showed cyclic variation. The slab response showed a different trend compared to that of the rail with the increasing frequency. Whereas the rail vibration magnitudes were reduced with increasing frequency from 200 to 300 Hz, the slab vibration responses increased in the same frequency range. The resonant frequency increased for both the rail and slab with the increasing stiffness. This suggested that their interaction should be incorporated to study the rolling noise generation.

The effects of the core damping material on the vibration response from the moving loads were analyzed as shown in Fig. 4. With the increasing damping ratio, the vibration magnitude was reduced especially at resonant frequencies. This frequency range corresponds to the mass-stiffness resonance region assuming the rail as the mass and the rail pad as the spring. In design process, the mass-stiffness resonant frequency is an important factor on estimating the effect of the rail pad damping. This damping is also influenced by the fastening system as well as the viscoelastic properties of the pads.

Fig. 3 Comparison of averaged velocity of (a) the rail and (b) the slab with change in the stiffness of the core damping material

Fig. 4 Effects of damping in the core material on the averaged velocity of (a) the rail and (b) the slab. The damping showed to have significant influence on the resonant responses
4. Conclusions

This study investigated the vibration of the sandwich structures excited by distributed moving loads. The effects of the core dynamic properties of the sandwich complex structure representing the railway tracks on the forced response were analyzed using the Rayleigh-Ritz method. Cross correlation functions for the displacement responses from the moving dynamic loads were applied to calculate response from the distributed loads. Through parametric studies of the moving velocity and the core damping properties, the important parameters having influence on the rolling noise generation were identified. The natural frequency increased and vibration of rail decreased with the increasing stiffness of the core damping material, but those of the slab increased. With the increasing damping in the core, the vibration responses of the rail and slab were reduced in the range near the mass-spring resonance frequency. The proposed analysis procedures are required when studying the vibration and noise reduction systems on complex railway structures equipped with various fastening and floating systems or functional concrete slabs.

References