Evaluation of Dynamic Properties of Trackbed Foundation Soil Using Mid-size Resonant Column Test

Yujin Lim†, Tien Hue Nguyen*, Seong Hyeok Lee** and Jin-Wook Lee**

Abstract
A mid-size RC test apparatus (MRCA) equipped with a program is developed that can test samples up to D=10 cm diameter and H=20 cm height which are larger than usual samples used in practice. Using the developed RC test apparatus, two types of crushed trackbed foundation materials were tested in order to get the shear modulus reduction curves of the materials with changing of shear strain levels. For comparison purpose, large repetitive triaxial compression tests (LRT) with samples of height H=60 cm and diameter D=30 cm were performed also. Resilient modulus obtained from the LRT was converted to shear modulus by considering elastic theory and strain level conversion and were compared to shear modulus values from the MRCA. It is found from this study that the MRCA can be used to test the trackbed foundation materials properly. It is found also that strain levels of $E_{v2}$ mostly used in the field should be verified considering the shear modulus reduction curves and proper values of $E_{v2}$ of trackbed foundation must be used considering the strain level verified.

Keywords : mid-size resonant column test, shear modulus, trackbed foundation materials, large repetitive load test, $E_{v2}$

1. Introduction
It is a well known fact that vibration induced by running trains causes deterioration of track foundation. Railway stiffness is a basic parameter of track design which influences the bearing capacity, the dynamic behavior of passing vehicles, track geometry quality and the life of track components [1]. The usual method to obtain the track stiffness is to run track loading vehicle in order to measure deflection under a wheel load size since the track stiffness is used as the parameter for calculating stresses in the elements of track and track foundation. The track stiffness is, however, a composite value that represents various aspects of stiffness of different materials involved in making foundation layers below track. In this study, in order to evaluate dynamic geotechnical properties of the trackbed foundation, a mid-size RC test apparatus equipped with an analyzing program is developed that can test samples up to D=10 cm diameter and H=20 cm height which is larger than usual samples of D=5 cm and H=10 cm used mostly in practice. The obtained dynamic properties such as shear modulus, $G$, of the sub-layers under track can be utilized to calculate track stiffness $k$ or composite equivalent modulus, $E_{equiv}$, of the sub-layers correctly that can consider the influences of strain and stress levels in each sub-layers of the track and can be used to determine formation thickness of the track. The larger the samples used in resonant column test, the more correct test G values in each layers can be obtained. Therefore, it is helpful to use mid-size resonant column test to get reliable data for shear modulus of the trackbed foundation materials by scrutinizing various factors affecting the shear modulus which contribute in determining composite track modulus $k$ and/or composite equivalent modulus, $E_{equiv}$. In this study, a proper way of determining composite equivalent modulus, $E_{equiv}$, by using shear modulus of the sublayers. The composite equivalent modulus, $E_{equiv}$, can be used as an input parameter for design of formation thickness.
2. Determination of Dynamic Properties and Composite Equivalent Modulus

2.1 Shear properties G

In general, shear modulus, G of a geomaterial such as trackbed foundation is strain dependent ([2]) as shown in Fig. 2.

In order to obtain correct design values of shear modulus, G, strain levels experienced by trackbed foundation materials in each layer must be known. The strain levels in the sublayers can be calculated by finite element analysis of track structure.

Resonant column (RC) testing has some advantages over the other techniques. A resonant column test apparatus (RCA) is capable of controlling shear strains induced in a soil specimen, and accurately measuring the resonant frequency of a specimen even at very small vibratory amplitude and allowing estimation of $G_{\text{max}}$ accurately without measuring applied torque. In addition, a RCA can widely vary the shear strain from very small to medium range (e.g. from $10^{-5}$% to $10^{-2}$%). A shear strain range produced by RC testing is similar to that produced by in-situ seismic testing. Therefore, a shear modulus and damping ratio measured using RC testing is comparable to that measured using geophysics surveying. Furthermore, RC testing is a non-destructive technique, which allows a soil specimen to be tested repeatedly at different effective stress levels.

2.2 Midsize resonant column test apparatus

In this study, a mid-size RC test apparatus (MRCA) equipped with analyzing program is developed that can test samples up to $D=10$ cm diameter and $H=20$ cm height which is larger than usual samples of $D=5$ cm and $H=10$ cm used mostly in practice. Thus, crushed stones with larger grains up to 38 mm in diameter used mostly in Korea as trackbed foundation materials in track construction could be considered effectively than conventionally used RC apparatus for evaluation of the dynamic properties of the materials by using the MRCA. The MRCA is designed and assembled based on the concept of fixed-free fixity conditions and driving mechanism proposed by Stokoe. The one end of the specimen is fixed to the rigid test base by the bottom pedestal while the other end is attached to drive plate (Fig. 3). Fig. 4 presents a schematic configuration of the MRCA used in this study.

A digital sinusoidal wave form is generated by Agilent 33220A 20 MHz Function/Arbitrary Waveform Generator, which is controlled by developed computer program via a NI PCI-6014 multifunction DAQ card, then amplified by an Eliezer EA100A power amplifier before going to the coils. The electro-magnetic force generated between the magnetic field of coils and the magnets mounted on drive plate cross arm, excites the specimen with the drive plate harmonic oscillation.

The motion of the specimen is monitored by the PCB343B51 accelerometer mounted on drive plate. Accelerometer signal goes through charge amplifier then con-
verted into digital signals by NI PCI-6014 card before being handled by the computer software in controlling PC.

Using the MRCA, three types of crushed stones used as trackbed foundation materials have been tested in order to obtain the dynamic properties of the trackbed foundation materials such as G/G_max reduction curves and damping ratio D.

### 2.3 Calibration of the MRCA

The drive plate mass polar moment of inertia, \( I_o \), is an important parameter in the analysis of RC test. In order to calculate the shear modulus, \( G \), Eq. 1 and 2 are used to obtain shear velocity, \( V_s \).

\[
\frac{I_s}{I_o} = \alpha \tan \alpha 
\]

(1)

\[
\alpha = \omega_s \frac{L}{V_s} 
\]

(2)

\( I_o \) can be calculated theoretically through geometry dimensions [3]. However, as shown in Fig. 4, drive system has complex shape with a magnet, an accelerometer, counterweight and threaded holes. Thus, it is difficult to calculate \( I_o \). Theoretical \( I_o \) was obtained as a reference value.

In experimental measurement of \( I_o \), three added masses and six calibration aluminum specimens are used. The drive plate calibration system can be modeled as a SDOF spring-mass system. The calibration specimen represents the massless spring with torsional spring stiffness, \( K_q \), and the top plate of calibration specimen and the drive plate represent the mass in the model. The natural resonant frequency, \( f_r \), can be expressed as:

\[
f_r = \frac{1}{2\pi} \sqrt{\frac{K_o}{I_o + \Delta I}} 
\]

(3)

where, \( f_r \) = resonant frequency (Hz), \( I_o \) = mass polar moment of inertia of drive plate system, \( I = \) mass polar moment of inertia of top plate of calibration specimen, \( K_q \) = torsional stiffness of calibration specimen. Eq. 3 has two unknowns, the calibration specimen torsional stiffness, \( K_q \), and mass polar moment of inertia of drive plate system, \( I_o \).

To obtain additional equations, the test was repeated on the same calibration specimen with already known mass polar moment of inertia of added mass, \( \Delta I \). In this study, to increase the precision of test results, three added masses were made for the calibration experiment. That means there are four equations (Eq. 3 to Eq. 6) with two unknowns:

\[
f_{o1} = \frac{1}{2\pi} \sqrt{\frac{K_o}{I_o + \Delta I_1}} 
\]

(4)

\[
f_{o2} = \frac{1}{2\pi} \sqrt{\frac{K_o}{I_o + \Delta I_2}} 
\]

(5)

\[
f_{o3} = \frac{1}{2\pi} \sqrt{\frac{K_o}{I_o + \Delta I_3}} 
\]

(6)

In free vibration, the resonant frequency, \( f_m \) is as follows:

\[
f_m = f_o \sqrt{1 - D^2} 
\]

(7)

where \( D \) = damping ratio of calibration specimen.

Substitution of Eq. 7 to Eq. 3 provides the following:

\[
f_n = \frac{1}{2\pi} \sqrt{\frac{K_o}{I_o + \Delta I}} \sqrt{1 - D^2} 
\]

(8)

Therefore, there can be four equations to solve for \( K_q \) and \( I_o \). Since \( D \) value for aluminum calibration bar is very small, the \( D \) value in Eq. 8 in this case of calibration using
aluminum bar is neglected. Table 1 and 2 explain in detail dimensions of calibration bars and added masses used for calibration process. The calibration procedure is explained in detail by Nguyen ([3]).

### 2.3 The Composite Equivalent Modulus

If the modulus of each sublayers of the trackbed foundation is known by using the MRCA and/or large repeated load test (LRT), the composite equivalent modulus, $E_{equiv}$, of trackbed foundation including ballast can be computed using the modulus and can be used as input design parameter for deciding of formation thickness.

In this study, the composite equivalent modulus, $E_{equiv}$, of trackbed foundation is defined as the follows:

$$E_{equiv} = \frac{E_B t_B + E_{SB} t_{SB} + E_{SG} D_{ESG}}{t_B + t_{SB} + D_{ESG}}$$

(9)

where $E_B$ = modulus of a ballast layer, $t_B$ = thickness of ballast layer, $E_{SB}$ = modulus of trackbed foundation layer, $t_{SB}$ = thickness of trackbed foundation layer, $E_{SG}$ = modulus of subgrade, $D_{ESG}$ = thickness or top influencing depth part of subgrade. The composite equivalent modulus, $E_{equiv}$, of trackbed foundation can be developed as an alternative design parameter instead of tack stiffness, $k$. In order to do this purpose, each modulus of the sublayers below the track must be evaluated using proper test procedure.

### 3. Test

#### 3.1 Properties of testing materials

Prior to the RC test, the typical trackbed foundation granular materials used in Korea were tested for obtaining basic physical properties. Grain size distribution and the test results are summarized in Table 1 and Table 2, respectively. Compaction test results are shown in Table 3. All materials tested are non-plastic (NP).

#### 3.2 Large Repetitive Triaxial Compression Test

Concept of shear stress ratio has been proposed ([4], [5]) because the trackbed materials, mostly composed of granular soil, are highly affected by shear strength of the material. In this study a large repetitive triaxial load test (LRT) has been adapted for performing test to get resilient modulus of the trackbed foundation materials. The test procedure which includes concept of shear stress ratio has been newly designed.

Permanent deformation of the granular material is dependent on strength since a limiting value of shear stress ratio $\phi/\tau_{max}$ is believed to control the permanent deformation. It is well known that the permanent deformation is highly dependent on the resilient modulus.

The shear strength of the granular material is governed by Mohr-Coulomb failure criteria. Therefore, decreased shear strength $\tau_{max}$ as a result of a lower friction angle would result in a higher shear stress ratio for the same stress level. Hence, the shear stress ratio becomes an indicator of the aggregate performance under various stress level. A combination of stress states was adapted in this study for the LRT procedure. Axial deviator stress calculation was performed by the following equations considering the stresses activated on the Mohr-Coulomb failure plane and by considering the Mohr-Coulomb failure criteria:

$$\frac{\tau}{\sigma_f} = \frac{\sqrt{\left(\sigma_f/2\right)^2 - (\sigma_f - \sigma_3 + \sigma_2/2)^2}}{c + \sigma_f \tan \phi}$$

(10)

$$\sigma_f = \frac{2 \sigma_2 + 2 \tan \phi \sigma_3 + 2 \tan \phi \sigma_4 - \sqrt{2 \sigma_2^2 + 4 \tan^2 \phi \sigma_3^2 + 4 \tan^2 \phi \sigma_4^2}}{2(1 + \tan^2 \phi)}$$

(11)

Using the same LRT apparatus, static triaxial compression tests were performed in order to get strength parame-
3.3 Test results

3.3.1 The normalized modulus reduction curve

Shear modulus reduction curves of trackbed foundation materials were obtained using the raw data from MRCA and small resonant column test apparatus (SRCA) and are shown in Fig. 6 through 9.

Normalized shear modulus ($G/G_{\text{max}}$) curves were obtained using the raw data from RC tests and are shown in Fig. 10 through Fig. 11. The normalized shear modulus ($G/G_{\text{max}}$) curves are compared with those offered by Seed et al. [6,7]. Three tested crushed stones all provide similar trend of the normalized shear modulus ($G/G_{\text{max}}$) values to the band suggested. All the normalized shear modulus ($G/G_{\text{max}}$) values are dependent on confining stress. The threshold strain increases with confining stress. Three

Table 4. Large static triaxial compression test results

<table>
<thead>
<tr>
<th>C (kPa)</th>
<th>$\phi$ (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trackbed fdn. Mat’l No.1</td>
<td>67</td>
</tr>
<tr>
<td>Trackbed fdn. Mat’l No.2</td>
<td>46</td>
</tr>
</tbody>
</table>

Table 5 Test conditions for large repeated load test (trackbed fdn. Mat’l No. 1)

<table>
<thead>
<tr>
<th>Confining stress</th>
<th>Shear stress ratio ($\tau/\tau_f$)</th>
<th>Deviatoric stress (kPa)</th>
<th>Number of repetition (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 kPa</td>
<td>0.30.7</td>
<td>86.25/249.96</td>
<td>5,000 10,000 total</td>
</tr>
<tr>
<td></td>
<td>0.50.7</td>
<td>159.26/249.96</td>
<td>5,000 10,000 total</td>
</tr>
<tr>
<td></td>
<td>0.70.7</td>
<td>249.96</td>
<td>10,000</td>
</tr>
<tr>
<td>69 kPa</td>
<td>0.30.7</td>
<td>124.39/360.49</td>
<td>5,000 10,000 total</td>
</tr>
<tr>
<td></td>
<td>0.50.7</td>
<td>229.69/360.49</td>
<td>5,000 10,000 total</td>
</tr>
<tr>
<td></td>
<td>0.70.7</td>
<td>360.49</td>
<td>10,000</td>
</tr>
</tbody>
</table>

Table 6 Test conditions for large repeated load test (trackbed fdn. Mat’l No. 2)

<table>
<thead>
<tr>
<th>Confining stress</th>
<th>Shear stress ratio ($\tau/\tau_f$)</th>
<th>Deviatoric stress (kPa)</th>
<th>Number of repetition (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 kPa</td>
<td>0.30.7</td>
<td>99.91/287.79</td>
<td>5,000 10,000 total</td>
</tr>
<tr>
<td></td>
<td>0.50.7</td>
<td>183.99/287.79</td>
<td>5,000 10,000 total</td>
</tr>
<tr>
<td></td>
<td>0.70.7</td>
<td>287.79</td>
<td>10,000</td>
</tr>
<tr>
<td>69 kPa</td>
<td>0.30.7</td>
<td>133.48/384.49</td>
<td>5,000 10,000 total</td>
</tr>
<tr>
<td></td>
<td>0.50.7</td>
<td>245.81/384.49</td>
<td>5,000 10,000 total</td>
</tr>
<tr>
<td></td>
<td>0.70.7</td>
<td>384.49</td>
<td>10,000</td>
</tr>
</tbody>
</table>
confining pressure is, the greater the shear modulus is at the same shear strain level.

It is found that the shear modulus decreases nonlinearly from threshold strain of 10-3%. The shear modulus obtained from mid-size specimen is bigger than that of small size specimen at the same shear strain level. Therefore, it is found that there is sample size effect on the shear modulus.

### 3.3.2 Resilient modulus

In this study large repetitive triaxial load tests (LRT) were performed using the same crushed stones which were compacted to 95% of maximum dry density (D=30 cm, H=60 cm) as the trackbed foundation materials used for the MRCA tests in order to compare the magnitude of maximum modulus quantitatively. Fig. 12 shows a typical test results from the LRT. All test results obtained from the LRT are summarized in Table 7 and Table 8.

![Fig. 9 Shear modulus reduction curves of No. 2 trackbed foundation material (SRCA, D=5 cm)](image)

![Fig. 10 Normalized shear modulus reduction curves of trackbed foundation material No. 1 (MRCA, D=10 cm)](image)

![Fig. 11 Normalized shear modulus reduction curves of trackbed foundation material No. 2 (MRCA, D=10 cm)](image)

![Fig. 12 Deviator stress-axial strain relation (confining stress: 35 kPa)- trackbed foundation material No.1](image)

<table>
<thead>
<tr>
<th>Shear stress ratio ((\tau/\tau_f))</th>
<th>0.3</th>
<th>0.7</th>
<th>0.5</th>
<th>0.7</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. load repetition</td>
<td>1st half</td>
<td>2nd half</td>
<td>1st half</td>
<td>2nd half</td>
<td>10,000</td>
</tr>
<tr>
<td>ER (MPa) ((\sigma_3=35) kPa)</td>
<td>167</td>
<td>180</td>
<td>176</td>
<td>154</td>
<td>148</td>
</tr>
<tr>
<td>ER (MPa) ((\sigma_3=69) kPa)</td>
<td>178</td>
<td>260</td>
<td>192</td>
<td>296</td>
<td>268</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shear stress ratio ((\tau/\tau_f))</th>
<th>0.3</th>
<th>0.7</th>
<th>0.5</th>
<th>0.7</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. load repetition</td>
<td>1st half</td>
<td>2nd half</td>
<td>1st half</td>
<td>2nd half</td>
<td>10,000</td>
</tr>
<tr>
<td>ER (MPa) ((\sigma_3=35) kPa)</td>
<td>246</td>
<td>297</td>
<td>260</td>
<td>324</td>
<td>260</td>
</tr>
<tr>
<td>ER (MPa) ((\sigma_3=69) kPa)</td>
<td>310</td>
<td>410</td>
<td>327</td>
<td>390</td>
<td>331</td>
</tr>
</tbody>
</table>
The resilient modulus obtained from the LRT are recalculated to get shear modulus, $G$, since shear modulus, $G$, can be converted to elastic modulus, $E$, easily by using the following equation:

$$G = \frac{E}{2(1+\nu)} \quad (12)$$

The following correlation equation can be affirmed using elasticity theory:

$$\gamma_{xy} - \gamma_3 = (1-\nu)\epsilon_v \quad (13)$$

Therefore, vertical compressive strain obtained from the LRT can be easily converted to shear strain using the above equation so that the converted shear modulus, $G_C$, the converted shear strain, $\gamma_C$, relation graphs are to be represented as shown in Fig. 13 and Fig. 14. The newly recalculated shear modulus values obtained from the LRT are compared to those values obtained from MRCA. Average values of the newly calculated shear modulus obtained from the LRT are shown also. As shown in Fig. 13, all shear modulus values obtained from the LRT are located in the range of medium to large strain ($\gamma > 0.01\%$). However, it is found the averaged values of shear modulus at each confining stress levels obtained from the LRT are close to maximum shear modulus, $G_{max}$ in the MRCA. This big difference in shear strain levels between LRT and MRCA is believed to be generated due to different measuring system. In order to get precise strain levels in LRT, it is required to use very accurate LVDT to measure strains in the range of small strain.

For design purposes, in order to determine correct values of modulus to be used in deciding the formation thickness, strain levels must be known from site field instrumentations and full 3-D finite element analysis considering dynamic amplification effects induced by running train on the track. In addition, it should be pointed out that the use of $E_{v2}$ which is defined as stiffness modulus obtained from unloading-reloading part of pressure-settlement curves from repeated plate bearing test (RPBT) mostly used in the field of track construction site to check degree of compaction and adapted as the design criteria value of trackbed foundation layer must be compared to those values as shown in Fig. 13 and Fig. 14 tested from the MRCA and the LRT since it is important to know the range of stain levels generated in the RPBT. Up to now, range of strain levels of repeated plate load test performed on the surface of compacted trackbed foundation layer and $E_{v2}$ test values accounted at these strain levels are not verified yet. This is going to be next task of this study.

4. Conclusion

In this study, a mid-size RC test apparatus (MRCA) equipped with program is developed that can test samples up to $D=10\text{ cm}$ diameter and $H=20\text{ cm}$ height which are larger than usual samples used mostly in practice. Using the developed RC test apparatus, two types of crushed trackbed foundation materials were tested in order to get the shear modulus reduction curves of the materials with changing of shear strain levels. For comparison purpose, large repetitive triaxial compression tests (LRT) with sample of height $H=60\text{ cm}$ and diameter $D=30\text{ cm}$ were performed also. Resilient modulus obtained from the LRT was converted to shear modulus by considering elastic theory and strain level conversion procedure and were compared to shear modulus values from the MRCA.

The following conclusions are made from this study:

1. The MRCA can be used to test the trackbed foundation materials properly. It is believed that even though the LRT can provide relatively larger values of shear modulus.
close to $G_{max}$ at shorter band of strain range and at medium to large strain levels, the MRCA can simulates generation of shear modulus with changing strain levels properly and relatively wider band of strain range.

(2) It is found that strain levels of $E_{v2}$ mostly used in the field should be verified considering the shear modulus reduction curves obtained from MRCA and/or the LRT and proper values of $E_{v2}$ of trackbed foundation. For design purposes, in order to determine correct values of modulus to be used in deciding the formation thickness, strain levels must be known from site field instrumentation and full 3-D finite element analysis considering dynamic amplification effects induced by running train on the track.

**References**


