Parametric Study of Thermal Stability on Continuous Welded Rail

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Abstract

The thermal buckling analysis of curved continuous welded rail (CWR) is studied for the lateral buckling prevention. This study includes a thermal buckling theory which accounts for both thermal and vehicle loading effects in the evaluation of track stability. The parameters include rail size, track lateral resistance, track longitudinal and torsional stiffnesses, initial misalignment amplitude and wavelength, track curvature, tie-ballast friction coefficient and truck center spacing. Parametric studies are performed to evaluate the effects of the individual parameters on the upper and lower critical buckling temperatures. The results show that the upper critical buckling temperature is highly affected by the uplift due to vehicle loads. This study provides a guideline for the improvement of stability for dynamic buckling in curved CWR track.

Keywords: Thermal stability, Lateral buckling, Continuous welded rail, Critical buckling temperature

1. Introduction

The previous railroad track used expansion joints between rails to avoid the expansion due to temperature increase. However, these joints weaken the connected portion, and make not only the noise but also the vibration. In order to prevent these problems, the expansion joints are removed and replaced with continuous welded rail (CWR) tracks which are welded and connected each other continuously. A substantial temperature increase in CWR tracks, however, will result in high stresses. These stresses can be dangerous since track buckling matheccur when the temperature reaches a critical value.

Kerr (1975) studied track buckling behavior of tangent CWR tracks. An improved analysis are performed static buckling temperature, including effects of the nonlinearity of longitudinal track resistance by Kerr (1980). Subsequently, theoretical and experimental studies on the track buckling temperature increase for tangent and curved tracks were performed (Samavedam et al., 1985). Recently, finite element method for the buckling analysis of CWR tracks have been proposed (Van, 1998).

In this study, analytic thermal buckling analysis of CWR tracks under vehicle loads are performed, based on static thermal buckling approach. The parameters used in this study include rail size, track lateral resistance, track longitudinal and torsional stiffnesses, initial misalignment amplitude and wavelength, track curvature, tie-ballast friction coefficient and truck center spacing. Parametric studies for the evaluation of quantitative effects of parameters and critical buckling temperatures have been performed varying one parameter at a time, and fixing other parameters at their nominal values.

2. CWR Buckling Theory

Figure 1 shows the buckling response characteristic which shows two critical buckling temperatures in general. At the upper buckling temperature $T_{bmax}$ the track will buckle out with zero disturbance or energy input. The lower buckling temperature $T_{bmin}$ means that the track will buckle out only with sufficient external energy input. Obviously, the difference between two critical temperatures and the energy required to buckle track need to be included in evaluating the stability for CWR track buckling (Samavedam et al., 1993). Sometimes, the buckling response, so called progressive buckling, shows...
that two critical temperatures coalesce into one, as shown in Figure 2.

For the numerical analysis in this study, the buckling track model is selected as shown in Figure 3. This model is simplified by assumption that ties and two rails move together when track buckle.

Figures 4 and 5 show the geometric shapes of simplified straight and curved CWR track model before and after the buckling. In simplified models, the bending stiffness of a tie and the torsional stiffness of the fasteners can be combined to a substitute torsional stiffness of fastener by (Van, 1998)

\[
\overline{\tau}_o = \frac{1}{1 + \frac{\phi}{6EI_z}}
\]

where \(\overline{\tau}_o\) is the combined torsional stiffness per fastener, \(\tau^*_o\) is the torsional stiffness per fastener, \(E,I_z\) is the bending stiffness of a tie, \(h_t\) is the length of a tie. Basic assumptions in the buckling analysis of CWR track are given as follows: The track domain is separated into two regions (a buckling zone and an adjoining zone), which are formulated by corresponding differential equations, respectively; A buckled zone where longitudinal displacement is neglected; An adjoining zone that extends to infinity where lateral displacement is neglected.

3. Development of Thermal Buckling Equations

3.1 Differential equation in buckled zone

The governing differential equation, in the buckled zone \((0 \leq \phi \leq \phi)\), can be derived by using the beam on the elastic foundation theory and then expressed as (Samavedam et al., 1993)

\[
\frac{EI_z}{R^4} \frac{d^4 w}{d\phi^4} + \left(\frac{P}{\overline{\tau}_o}\right) \frac{d^2 w}{d\phi^2} = -F(\phi) + \frac{\overline{P}}{R} \frac{d^2 w}{d\phi^2}
\]

where \(w(\phi)\) is the lateral displacement, \(E\) is the modulus of elasticity, \(I_z\) is the moment of inertia of two rails for lateral bending (i.e., about the vertical axis), \(\overline{P}\) is the compressive
load, \( F[w(0)] \) is the lateral resistance distribution function, \( \tau_v \) is the linear torsional stiffness of fasteners per unit track length and \( w_v(0) \) is the initial imperfection distribution. The solution of Eq. (2) can expressed as the following equations.

\[
w(0) = \sum_{m = 1, 3, 5, \ldots}^{\infty} A_m \cos \left( \frac{m\pi 0}{2\phi} \right)
\]

\[
\frac{d^2 w}{d\theta^2} = \sum_{m = 1, 3, 5, \ldots}^{\infty} b_m \cos \left( \frac{m\pi 0}{2\phi} \right)
\]

\[
F[w(0)] = \sum_{m = 1, 3, 5, \ldots}^{\infty} a_m \cos \left( \frac{m\pi 0}{2\phi} \right)
\]

Substituting Eqs. (3)–(4) into Eq. (2) results in

\[
A_m = \frac{EI_L \left( \frac{m\pi V}{2\phi} \right)^2 \left( \frac{\bar{P} - \tau_v}{R^2} \right) \left( \frac{m\pi}{2\phi} \right)^2}{R^2 \phi^2}
\]

Applying boundary conditions, \( w' = 0 \) at \( \theta = \pm \phi \), the following equation is obtained.

\[
w'(\phi) = \sum_{m = 1, 3, 5, \ldots}^{\infty} mA_m \sin \left( \frac{m\pi 0}{2\phi} \right) = 0
\]

The compressive load \( \bar{P} \) is obtained by iteration scheme from Eq. (8) with the assumed buckling length \( L \).

### 3.2 Differential equation in adjoining zone

If the longitudinal resistance is assumed to be proportional to the longitudinal displacement, the governing differential equation in adjoining zone (\( \theta > \phi \)) from equilbrium considerations in the longitudinal direction can be written as

\[
AE \frac{\partial^2 u}{\partial \theta^2} = k_v u
\]

where \( u(\theta) \) is the longitudinal displacement, \( A \) is the cross sectional area of two rails, and \( k_v \) is the longitudinal stiffness. The solution of Eq. (9) is obtained by the condition \( u = u' = 0 \) in the limit as \( x \) approaches infinity:

\[
u(x) = Ce^{-P x} \quad \text{or} \quad u = -\psi u
\]

where \( \Psi^2 = k_v/AE \)

### 3.3 Calculations of vehicle effects on the lateral stability of CWR track

Quasi-static loads model, as shown in Figure 6, is assumed to determine loss and addition of lateral resistance due to the vertical track deformation under wheel loads. The differential equation for the vertical deflection \( \nu \) can be written as

\[
EI_{yy} \frac{\partial^4 \nu}{\partial x^4} = \sum \delta(\theta - 0) V_\nu \cos \theta + Q
\]

where \( EI_{yy} \) is the flexural rigidity for two rails in the vertical plane, \( k_v \) is the track foundation stiffness, \( V_\nu \) are the vertical wheel loads, \( \delta_\nu \) are the dirac delta functions, \( Q \) is the track weight per unit length. The solution of Eq. (11) is given by

\[
\nu = \nu_o + \nu_1 = \frac{Q}{k_v} + \frac{1}{2k_v} \int F_\beta (\beta x) \sin \beta x e^{-\beta x}
\]

The lateral resistance \( F \) in Eq (1) is divided by three components, as shown following

\[
F_{\text{static}} = F_s + F_z + F_x
\]

\[
F_{\text{dynamic}} = \left( F_{\text{static}} + \mu k_v \nu_1(x) \right) \frac{1}{F_{\text{static}}} \text{For } \nu < 0 \text{ (uplift)}
\]

\[
F_{\text{dynamic}} = \left( F_{\text{static}} + \mu k_v \nu_1(x) \right) \text{otherwise}
\]

where \( \mu \) is the friction coefficient. In Eq. (14), \( F \) is divided into two case, considering loss and addition of resistance due to the vertical deformation under wheel loads.

### 3.4 The fourier coefficients of governing differential equation

The Fourier coefficient \( a_m \) accounts for the lateral resistance contribution. In the case where no uplift occur, The Fourier coefficient \( a_m \) can be determined as

\[
a_m = a_{m1} + a_{m2} = \frac{\mu k_v}{AE} \int \sin \left( \frac{m\pi \nu}{2\phi} \right) d\theta + \frac{2\mu k_v}{\phi} \int \nu_1 \cos \left( \frac{m\pi \nu}{2\phi} \right) d\theta
\]
In uplift zone, the $a_{m2}$ in Eq. (15) is replaced by

$$a_{m2} = \frac{2}{\phi_o} F_m \cos\left(\frac{m\pi \delta}{2\phi_o}\right)$$  \hspace{1cm} (16)

If lateral loads are considered, the solution of the Fourier coefficient $a_m$ is again affected. In these case, the Fourier coefficient $a_m$ becomes

$$a_m = a_{m1} + a_{m2} + a_{m3}$$  \hspace{1cm} (17)

where $a_{m3}$ term is the lateral loading contribution. If lateral Loads are modeled as shown Figure 7, $a_{m1}$ is given by

$$a_{m3} = \begin{cases} 0 & \text{For } L \leq S_1 \\ \frac{2F_m}{L} \cos\left(\frac{m\pi S_1}{2L}\right) & \text{For } S_1 < L \leq S_2 \\ \frac{2F_m}{L} \left[ \cos\left(\frac{m\pi S_2}{2L}\right) + \cos\left(\frac{m\pi S_1}{2L}\right) \right] & \text{For } L > S_2 \end{cases}$$  \hspace{1cm} (18)

If the initial imperfection shape is assumed as,

$$w_o(\theta) = \delta_0 \left[1 - 2 \left(\frac{0}{\phi_o}\right)^2 + \left(\frac{0}{\phi_o}\right)^4\right]$$  \hspace{1cm} (19)

The Fourier coefficient that accounts for the effect of initial imperfection in the track is derived as

$$\frac{h_m}{R^2} = -\frac{16\delta_0}{m\pi L_o} \left[1 - 3\left(\frac{L_o}{L_o}\right)^2 \left(1 - 2\frac{2}{m\pi}\right)^2\right] \sin\left(\frac{m\pi}{2}\right)$$  \hspace{1cm} (20)

$$\text{For } \phi \leq \phi_o$$

$$\frac{h_m}{R^2} = -\frac{16\delta_0}{m\pi L_o} \left[ -6\left(\frac{L_o}{L_o}\right)^2 \left(\frac{2}{m\pi}\right) \cos\left(\frac{m\pi L_o}{2L_o}\right) \right.$$

$$+ 2 \left[-1 + 3\left(\frac{2L_o}{m\pi L_o}\right)^2\right] \sin\left(\frac{m\pi L_o}{2L_o}\right) \right] \text{ For } \phi > \phi_o$$  \hspace{1cm} (21)

where $L_o$ is the misalignment half-wavelength and $\phi_o = L_o/R$.

Also, $c_m = \frac{4}{m\pi} \sin\left(\frac{m\pi}{2}\right)$  \hspace{1cm} (22)

### 3.5 Temperature Calculations

The temperature equation is derived by using continuity requirements on the longitudinal displacement between the buckled and adjoining zones, and expressed as following equation.

$$U(\theta) = -P \frac{R \phi}{AE} - Z + aT R \phi$$  \hspace{1cm} (23)

where $a$ is the coefficient of thermal expansion and $T$ is the temperature change, and $Z$ is the apparent shortening of rail and is written as (Allen and Bulson, 1980)

$$Z = \int_{0}^{\delta} \left[ \frac{w^2}{2R^2} + \frac{w'_o^2}{R^2} \right] R d\theta$$  \hspace{1cm} (24)

and the solution is expressed by an infinite series.

$$Z = \sum_{m=1,3,5}^{\infty} \left[ \frac{2L_o m \pi}{m \pi R} A_m \sin\left(\frac{m \pi}{2}\right) \frac{A_m^2}{2L_o^2} \right]$$  \hspace{1cm} (25)

Substituting Eq. (23) into Eq. (10), the following equation is obtained.

$$T = \frac{P}{AE} + \frac{Z \psi}{\alpha(1 + \psi L)}$$  \hspace{1cm} (26)

### 4. Parametric Study

Parametric studies on the lateral resistance $F$, the longitudinal stiffness $k_l$, the torsional stiffness $k_t$, the initial misalignment amplitude $\delta_o$, the initial misalignment half-wave length $L_o$, track curvature and tie-ballast friction coefficient $\mu$, and truck center spacing $D_t$ are performed in the following to evaluate the quantitative effects of parameters and critical buckling temperatures. In parametric studies, the method, which are changing one parameter at a time, fixing others on their nominal value, is selected to investigate the effect of the individual parameter. Unless otherwise noted, the fixed values for the parameters are: rail size 60 kg; degree of curvature = 5°; $F = 700$ kg/$m$; $k_f = 40$ kg/cm$^2$; $\tau_o = 0$; $\delta_o = 2.5$ cm; $L_o = 600$ cm; $E = 2.1 \times 10^6$ kg/cm$^2$; $\mu = 0.6$, $F_V = 17$ ton, $F_f/F_V = 0$; $k_V = 200$ kg/cm$^2$; $D_1 = 10$ m, $D_2 = 180$ cm, $\alpha = 1.15 \times 10^{-5}$ 1/°C. Subsequently, additional works are also performed to compare dynamic buckling with static thermal buckling. analyses for comparison are performed using two vehicle load cases, which are modeled by T.G.V. train.

#### 4.1 Effects of rail size

Table 1 shows the size of rails that are presently used in Korea. The results for effect of rail size on buckling are shown in Fig. 8 for rail sizes in Table 1. The results show that buckling temperature decrease with increasing in rail
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The bending moment of inertia and cross-sectional area increase with increasing rail size. The increase in area increases the thermal force, which offsets the corresponding increase in bending stiffness, thus reducing the overall buckling strength. The upper buckling temperature is more sensitive to the rail size.

4.2 Effects of track curvature

The effects of track curvature are studied using curves ranging from 0° to 9°. The buckling results are shown in Figure 9. The results show that increasing curvature reduces both the upper and the lower buckling temperatures \( T_{b,max} \) and the upper buckling is more sensitive to the changes in curvature. For high curved tracks, the buckling temperatures are drastically reduced in comparison to low curved tracks. In this study, the progressive buckling occurs in curvature of 8 and higher.

4.3 Effects of lateral resistance \( F \)

The lateral resistance exerted by the ballast on the rail-tie structure consists of the friction force between the ballast and the bottom surface \( F_b \) and the two long side of ties \( F_e \), as well as of the pressure the ballast exerts against the end surface of ties \( F_c \). The lateral resistance to the lateral displacement is nonlinear (Kerr, 1980), but it is needed extensive experimental work, so not considered in these analyses. Studies are performed by assuming the ratio of components as \( F_b : F_e : F_c = 2.5 : 5 : 2.5 \). The effects of lateral resistance are shown in Figure 10. The values of \( F = 500, 600, 700, 800, 900, 1000 \) are used. As \( F \) increases, both the upper and the lower buckling temperatures increase, and the upper buckling temperature \( T_{b,max} \) is more sensitive than the lower buckling temperature to the changes in lateral resistance. The results show that the type of ballast and consolidation levels, which are characterized by the track resistance, are important parameters for buckling strength.

Table 1. Typical Rail Properties

<table>
<thead>
<tr>
<th>Size</th>
<th>Area (cm²)</th>
<th>Weight (kg/m)</th>
<th>Area Moment ( I_x ) (cm⁴)</th>
<th>Area Moment ( I_y ) (cm⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30A</td>
<td>38.26</td>
<td>30.1</td>
<td>604</td>
<td>152</td>
</tr>
<tr>
<td>37A</td>
<td>47.28</td>
<td>37.2</td>
<td>952</td>
<td>227</td>
</tr>
<tr>
<td>50N</td>
<td>64.05</td>
<td>50.4</td>
<td>1,960</td>
<td>322</td>
</tr>
<tr>
<td>60</td>
<td>77.50</td>
<td>60.8</td>
<td>3,090</td>
<td>512</td>
</tr>
</tbody>
</table>

Fig. 8 Buckling curve varying rail size

Fig. 9 Buckling curve varying degree of curvature

Fig. 10 Buckling Curve varying lateral resistance
4.4 Effects of longitudinal stiffness $k_l$

Track longitudinal resistance is the resistance which was offered by ties and ballast to the rails. The effects of longitudinal resistance are examined for the values of $k_l = 25, 30, 35, 40, 45, 50, 55, 60 \text{ kg/cm}^2$. The buckling results are shown in Figure 11. The lower buckling temperature shows a slight increase with the increase of longitudinal stiffness, whereas the upper buckling temperature is essentially independent of changing stiffness. There is a precaution in this study that original longitudinal resistance response is non-linear, but the longitudinal resistance response is assumed as linear, because the small longitudinal displacement occurs during buckling (usually less than 0.5 cm) (Kerr, 1980).

4.5 Effects of torsional stiffness of fastener $\tau_o$

Torsional resistance is exerted on the rail by the fasteners. Parameter $\tau_o$ used means combined torsional stiffness per fastener as stated earlier, and $\tau_o$ in Eq. (3.1) can be obtained from $\tau_o = 2\tau_o/d$, and $d$ is 60 cm for the tie space. The effects of fastener torsional resistance on the CWR track buckling are examined using $\tau_o$ ranging from 0 to 1800 $t \cdot \text{cm/rad}$. The results are shown in Figure 12. Both buckling temperatures increase with increasing torsional stiffness. The lower buckling temperature is more sensitive to the stiffness change, resulting in a significant increase.

4.6 Effects of initial misalignment amplitude $\delta_o$

Misalignment effects are examined first by using typical misalignment amplitudes ranging from 1.5 to 4.5 cm. The results are shown in Figure 13. Both temperature quantities decrease as the misalignment amplitude increase, with the upper critical temperature being more sensitive to these changes. A progressive buckling condition appears at a amplitude of approximately 4.5 cm.

4.7 Effects of Initial Misalignment Wave-Length $L_o$

The misalignment effects are studied using constant misalignment amplitude of 2.5 cm, but with changing misalignment wave length ranging from 300 to 900 cm. The results are shown in Figure 14. The lower critical temperature is relatively insensitive to the effects of wave-length. Similarly to the effects of curvature, a progressive buckling condition appears at a wave length of 400 cm. The difference between upper and lower temperature is getting larger when the wave length increases.
4.8 Effects of tie-ballast friction $\mu$

The tie bottom surface roughness is an important parameter as it determines the component of the base resistance $F_b$. The roughness factor is artificially expressed as friction coefficient, which is influenced by the type of ties and ballast, the age of tie, and amount of consolidation. The effects of friction coefficient on buckling are examined using ranging 0.4 to 0.8. The result are shown in Figure 15. As expected, the increasing surface roughness of the tie bottom (increasing $\mu$) increases both the upper and the lower buckling temperatures. The increase of buckling temperatures is less than 5°C over the range studied here.

4.9 Effects of the truck center spacing

The effects of vehicle truck center spacing as shown Figure 4.1 are examined using a range of 600 to 1800 cm. Each of these cars has four axles with an axle spacing of 180 cm. The results are shown Figure 16. The buckling temperatures do not follow the same trends with increasing truck center spacing. The upper buckling temperature is more sensitive to truck center spacing change, first decreases, but then increases slightly. Also, it can be seen that a critical truck center spacing is about 11 m. This spacing is considered to be critical because the difference between upper buckling temperature and lower buckling temperature is small. Therefore, the buckling can occur with a little energy.

5. Conclusions

This study develops a thermal buckling theory for the evaluation of curved CWR track buckling and the relationship between the critical temperatures and the required buckling energy used as a measure of the degree of stability. Parametric studies are performed to evaluate the effects of rail size, track curvature, track lateral resistance, track longitudinal and torsional stiffnesses, initial misalignment amplitude and wavelength, tie-ballast friction coefficient and truck center spacing. The following conclusions are listed.

(1) The results show that the upper and lower critical temperatures both decrease with increasing rail size.

(2) Curvature in general is one of the most important parameter which influence buckling strength. Both the upper and the lower buckling temperatures decrease with increasing curvature, and the upper buckling temperature is more sensitive. The progressive buckling occurs in curvature of $8^\circ$ and higher.
(3) The lateral resistance is an important parameter, which influences buckling temperatures. The buckling temperatures increase as the lateral resistance increases.

(4) The longitudinal stiffness has a minimal influence on the upper buckling temperature, whereas the lower buckling temperature shows a slight increase with increasing stiffness.

(5) The torsional stiffness significantly influences the lower buckling temperature. In this study, an increase of 12°C can be achieved by increasing torsional stiffness. The influence on the upper buckling temperature is very small.

(6) Misalignment significantly influences the lower critical temperature as well as the upper critical temperature. Therefore, good alignment is very important to prevent the CWR track buckling.

(7) As tie ballast friction coefficient increases, both the upper and the lower buckling temperatures increase. Over the range studied here, the increase of buckling temperatures is less than 5°C.

(8) The buckling temperatures do not follow the same trends with increasing truck center spacing. Results show that a critical truck center spacing is about 11 m. Truck center spacing most strongly influences the upper buckling temperature, with the lower buckling temperature being relatively unaffected.

### Reference